A SUPPLY SIDE THEORY OF MEDIATION
APPENDIX

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A.1. Some Preliminary Functions, from Kydd (2003)

Basic concepts (based on Kydd's (2003) Table 1)

Term and definition	Range
s = status quo division of the issue	[0, 1]
p = Player 1's chance of winning a conflict	[0, 1]
x = Player 1's proposal for his share of the issue	[0, 1]
$y = 1-x$ = Player 2's share of the issue	[0, 1]
k = upper bound on Player 2's cost of fighting	positive
ε = likelihood that the signal from Nature is in error	(0, 0.5)
c_i = player i's cost of conflict	positive, c_2 bound by k
h = likelihood Player 2 has high costs	[0, 1]
h_H = likelihood Player 2 has high costs given an H signal	[0, 1]
h_L = likelihood Player 2 has high costs given an L signal	[0, 1]
A, B = boundaries of k between which Player 1's offer depends on k	positive

Player 1's beliefs:

$$h = \frac{p+k-s}{k}, \; h_H = \frac{(1-\epsilon)h}{(1-\epsilon)h+\epsilon(1-h))}, \; h_L = \frac{h\epsilon}{h\epsilon+(1-\epsilon)(1-h)}$$

Offers made by Player 1:

$$x_H = \frac{\frac{1-\epsilon}{\epsilon}(p+k-s)+s+p-c_1}{2}, \text{ or } x_L = \frac{\frac{\epsilon}{1-\epsilon}(p+k-s)+s+p-c_1}{2}$$

Possible ranges for k, the upper bound on Player 2's cost of fighting:

$$A_H = s - p + \frac{\epsilon}{1-\epsilon}(p-s+c_1), \; B_H = s - p + \frac{\epsilon}{1-\epsilon}(s-p+c_1)$$
$$A_L = s - p + \frac{1-\epsilon}{\epsilon}(p-s+c_1), \; B_L = s - p + \frac{1-\epsilon}{\epsilon}(s-p+c_1)$$

A.2. Introducing b(S_n-S_m) to the mediator's utility function

In Kydd's original model, if conflict is avoided, the Mediator's utility is βx, where x is Player 1's offer, which is accepted by Player 2. And if conflict occurs, the Mediator's utility is *βx-c_m*.

In our extension of Kydd's model, if conflict is avoided and the Mediator tells the truth, then her utility is still βx. But if conflict is avoided and the Mediator lies, her utility is $\beta x - b(S_n - S_m)$, where b is the probability of getting being caught in a lie, S_n is the signal from Nature, and S_m is the signal the Mediator communicates to Player 1. If conflict occurs and the Mediator tells the truth, she receives $\beta x - c_m$, as in Kydd's model. But if conflict occurs and the Mediator lies, she receives $\beta x - b(S_n - S_m) - c_m$.

A.3 Calculating the Mediator's Decisions

Case 1: $k < A_H$

Whether Nature sends the H signal (Player 2 has high costs) or the L signal (Player 2 has low costs), Player 1 will offer its maximum, p, no matter what his beliefs are, and Player 2 will accept. So the Mediator gets bp if it tells the truth (because $S_n = S_m$) and $\beta p - b(S_n - S_m)$ if she lies. Unless $b = 0$, the Mediator is honest. This is different from Kydd's result, in which the Mediator is indifferent and babbling equilibria mix with truth-telling.

Case 2: $k \in [A_H, \min\{A_L, B_H\}]$

2.1: Nature Sends H signal to the Mediator ("Player 2 Has High Costs")

Utilities for the Mediator, for both telling the truth ($U_{2.1t}$) and lying ($U_{2.1l}$):

$U_{2.1t} = (1 - (h_H + (s - x_H)(\frac{1-h_H}{s-p})))(-c_m)$

$U_{2.1l} = -b(S_n - S_m)$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$b^* = -\frac{(k(\epsilon-1)+(p-s)(2\epsilon-1)+c_1\epsilon)c_m}{2(k(\epsilon-1)+(p-s)(2\epsilon-1))(S_m-S_n)}$

Note that b^* is positive when $\frac{k(\epsilon-1)+(p-s)(2\epsilon-1)}{\epsilon} < c_1$, which is always true because $0 < \epsilon \leq 0.5$.

The Mediator tells the truth whenever $b > b^*$; when the international system is more democratic, when the potential mediator is democratic, or when the disputants' shared IO memberships increases, this increases b, improving the chances that this condition is met.

Case 2.2: Nature Sends L signal to the Mediator ("Player 2 Has Low Costs")

Utilities for the Mediator:

$U_{2.2t} = 0$

$U_{2.2l} = (h_L + (s - x_H)(\frac{1-h_L}{s-p}))(-b(S_n - S_m)) + (1 - (h_L + (s - x_H)(\frac{1-h_L}{s-p})))(-c_m - b(S_n - S_m))$

Telling the truth is always better than lying in this case ($U_{2.2l} < 0$). Since $\beta = 0$, there is no incentive to lie. But since Player 1 cannot see the signal from Nature, he can only conclude that the Mediator is credible when $b > b^*$, therefore the Mediator is only credible when the condition for truth-telling in Case 2.1 is also met.

Case 3: $k \in [A_L, B_H]$

3.1: Nature Sends H signal to the Mediator ("Player 2 Has High Costs")

Utilities for the Mediator:

$U_{3.1t} = (1 - (h_H + (s - x_H)(\frac{1-h_H}{s-p})))(-c_m)$

$U_{3.1l} = (h_H + (s - x_L)(\frac{1-h_H}{s-p}))(-b(S_n - S_m)) + (1 - (h_H + (s - XL)(\frac{1-h_H}{s-p})))(-c_m - b(S_n - S_m))$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$b^* = \frac{(k+p-s)(2\epsilon-1)c_m}{2(\epsilon-1)(k(\epsilon-1+(p-s)(2\epsilon-1))(S_m-S_n)}$

For the sake of simplification, let $F_t = (h_H + (s - x_H)(\frac{1-h_H}{s-p}))$. Because F_t is the probability of Player 2 accepting Player 1's offer given that Nature has sent H to the Mediator and the Mediator has sent H to Player 1, we know that $0 < F_t < 1$.

Also, let $F_t = (h_H + (s - x_L)(\frac{1-h_H}{s-p}))$. Because F_l is the probability of Player 2 accepting Player 1's offer given that Nature has sent L and the Mediator has sent H to Player 1, we know that $0 < F_l < 1$. The revised utilities for the Mediator:

$U^r_{3.1t} = (1 - F_t)(-c_m)$

$U^r_{3.1l} = F_l(-b(S_n - S_m)) + (1 - F_l)(-c_m - b(S_n - S_m))$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$b^* = \frac{c_m(F_t - F_l)}{S_m - S_n}$

As long as $F_l > F_t$, b^* is positive. Because $x_H > x_L$, we know that $F_l > F_t$ is always the case, so b^* is always positive The Mediator tells the truth whenever $b > b^*$; when the international system is more democratic, when the potential mediator is democratic, or when the number of disputants' shared IO memberships increases, this increases b, improving the chances that this condition is met.

3.2: Nature Sends L signal to the Mediator ("Player 2 Has Low Costs")

Utilities for the Mediator:

$U_{3.2t} = (1 - (h_L + (s - x_L)(\frac{1-h_L}{s-p})))(-c_m)$

$U_{3.2l} = (h_L + (s - x_H)(\frac{1-h_L}{s-p}))(-b(S_n - S_m) + (h_L + (s - x_L)(\frac{1-h_L}{s-p})))(-c_m - b(S_n - S_m))$

Again, we make substitutions for probabilities with complex forms:

$$U_{3.2t}^r = (1 - G_t)(-c_m)$$
$$U_{3.2l}^r = G_l(-b(S_n - S_m)) + (1 - G_l)(-c_m - b(S_n - S_m))$$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$b^* = \frac{c_m(G_t - G_l)}{S_m - S_n}$ This is identical to the result in 3.1.

Case 4: $k \in [B_H, A_L]$

4.1: Nature Sends H signal to the Mediator ("Player 2 Has High Costs")

Utilities for the Mediator:
$$U_{4.1t} = (1 - h_H)(-c_m)$$
$$U_{4.1l} = -b(S_n - S_m)$$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$$b^* = \frac{c_m(h_H - 1)}{S_m - S_n}$$

We know that $h_H \leq 1$, so b* is always at least zero. The Mediator tells the truth whenever $b > b^*$; when the international system is more democratic, when the potential mediator is democratic, or when the number of disputants' shared IO memberships increases, this increases b, improving the chances that this condition is met.

4.2: Nature Sends L signal to the Mediator ("Player 2 Has Low Costs")

Utilities for the Mediator:
$$U_{4.2t} = 0$$
$$U_{4.2l} = h_L(-b(S_n - S_m)) + (1 - h_L)(-c_m - b(S_n - S_m))$$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$$b^* = \frac{c_m(h_L - 1)}{S_n - S_m}$$

Because $h_L < 1$ and $Sn > Sm$, b* is always negative. Therefore, there is no b that makes lying better than truth-telling. The mediator is always honest under this condition. Although the Mediator has no incentive to lie, Player 1 cannot see the signal from Nature and can only conclude that the Mediator is credible when $b > b^*$, therefore the Mediator is only credible when the condition for truth-telling in 4.1 is also met.

Case 5: $k \in [\max\{A_L, B_H\}, B_L]$

5.1: Nature Sends H signal to the Mediator ("Player 2 Has High Costs")

Utilities for the Mediator:
$U_{5.1t} = (1 - h_H)(-c_m)$
$U_{5.1l} = (h_H + (s - x_L)(\frac{1-h_H}{s-p})(-b(S_n - S_m))) + (1 - (h_H + (s - x_L)(\frac{1-h_H}{s-p})(-c_m - b(S_n - S_m)))$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$b^* = \frac{c_m(h_H - 1)(s - x_L)}{(p-s)(S_n - S_m)}$

We know that $s > p$ and $S_n > S_m$, so the denominator is negative. We also know that $c_m > 0$ and $0 \leq h_H \leq 1$. Therefore, if $s > x_L$, then any b will be larger than b^*, and the Mediator will tell the truth. If $s < x_L$, the Mediator will tell the truth if $b > b^*$; when the international system is more democratic, when the potential mediator is democratic, or when the number of disputants' shared IO memberships increases, this increases b, improving the chances that this condition is met.

5.2: Nature Sends L signal to the Mediator ("Player 2 Has Low Costs")

Utilities for the Mediator:
$U_{5.2t} = (1 - (h_L + (s - x_L)(\frac{1-h_L}{s-p})))(-c_m)$
$U_{5.2l} = (h_L(-b(S_n - S_m))) + (1 - (h_L)(-c_m - b(S_n - S_m)))$

Setting these two utilities equal to each other and solving for b yields a critical value, b^*:

$b^* = \frac{c_m(h_H - 1)(s - x_L)}{(p-s)(S_m - S_n)}$

This is similar to the result in 5.1, except that we now know the denominator is positive. Therefore, if $s < x_L$, then any b will be larger than b^*, and the Mediator will tell the truth. If $s > x_L$, the Mediator will tell the truth if $b > b^*$; when the international system is more democratic, when the potential mediator is democratic, or when the number of disputants' shared IO memberships increases, this increases b, improving the chances that this condition is met.

Case 6: $k > B_l$

The signals from Nature and the Mediator are irrelevant. Utilities for the Mediator:

$U_{6.1t} = (1 - h)(-c_m)$
$U_{6.1l} = h(-b(S_n - S_m)) + (1 - h)(-c_m - b(S_n - S_m))$

Setting these two utilities equal to each other and solving for b yields a critical value, b^* = 0. Lying is dominated by telling the truth. Babbling equilibria can occur when $b = 0$.